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### **ESTIMATION OF REGIONAL SNOW DEPTH CORRELATION SCALES AND IMPLICATIONS FOR GLOBAL SNOW ANALYSES**

**Cezar Kongoli<sup>1,2</sup>, Thomas M. Smith<sup>2</sup>**

<sup>1</sup>*Earth System Science Interdisciplinary Center (ESSIC), University of Maryland College Park;*  
<sup>2</sup>*National Oceanic and Atmospheric Administration (NOAA), National Environmental Satellite, Data,  
and Information Service (NESDIS), College Park, MD, United States;*

*Corresponding author: [Cezar.Kongoli@noaa.gov](mailto:Cezar.Kongoli@noaa.gov);*



# Estimation of regional snow depth correlation scales and implications for global snow analyses

*Cezar Kongoli and Thomas M. Smith*

*Earth System Science Interdisciplinary Center (ESSIC),  
University of Maryland College Park, USA*

**NOAA/NESDIS, College Park, Maryland, USA**

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# OVERVIEW

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- ❖ **STATISTICAL INTERPOLATION FOR SNOW DEPTH ANALYSIS**
- ❖ **NEW STUDY\*** (*Kongoli and Smith, Front. Earth Sci., 2023, volume 11*)
  - ❖ **Measured snow depth spatial correlations over North America**
  - ❖ **Fitted correlation functions and scale parameter estimates**
  - ❖ **Main findings**
- ❖ **IMPLICATIONS FOR OPERATIONAL SNOW DEPTH ANALYSIS**

*\* Modelling and estimation of snow depth spatial correlation structure from observations over North America”*

# STATISTICAL/OPTIMAL INTERPOLATION FOR SNOW DEPTH

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## ❖ Introduced by Brasnett (1999)

*Brasnett, B., 1999. A global analysis of snow depth for numerical weather prediction. J. Appl. Meteor., 38, 726-740.*

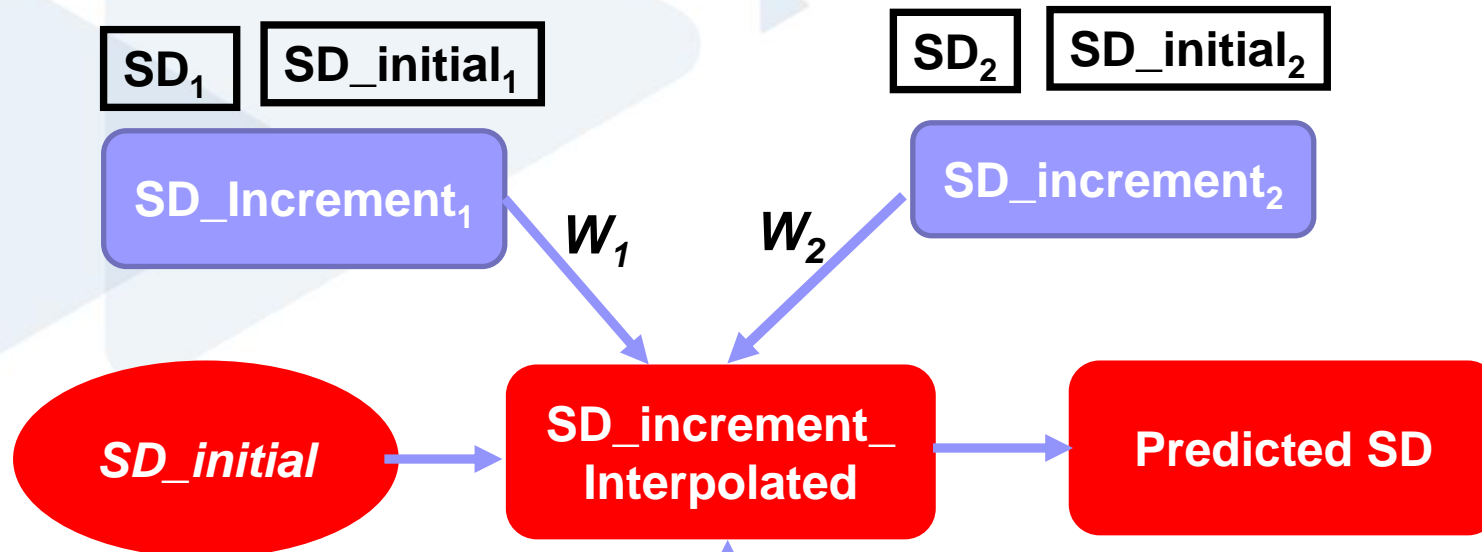
❖ Uses station-measured snow depth observations to generate daily gridded snow depth maps over the globe at 24 km resolution

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***Kriging Interpolation Method***, adapted for use in meteorology by L.S.

***Gandin, 1965.*** “Objective Analysis of Meteorological Fields”, Israel Program for Scientific Translation (from Russian).

# STATISTICAL/OPTIMAL INTERPOLATION FOR SNOW DEPTH (SD) EXAMPLE USING ONLY THREE MEASUREMENTS



➤  $W_1$ ,  $W_2$ , and  $W_3$  are the spatial weights calculated from linear equations with coefficients the spatial correlations among pairs of stations/interpolation grid cell

➤ Snow depth increment is the difference between the measured snow depth and an initial guess snow depth

➤  $SD\_increment\_interpolated$  is calculated as the weighted average of the snow depth increments at stations 1, 2, and 3.

# Snow Depth Optimal Interpolation (OI)

- ❖ **SD** increment *at analysis point*  $k$   $\Delta\mathbf{SD}_k$  is computed as the weighted average of observed increments  $\Delta\mathbf{SD}_i$  surrounding  $k$ :

$$\Delta\mathbf{SD}_k = \sum_{i=1}^N w_i \Delta\mathbf{SD}_i$$

$\Delta\mathbf{SD}_i$  is the difference between the **observed SD** and the **first guess SD** at each observation point  $i$  [ $i = 1, N$ ]

- ❖ The vector of optimum weights at  $k$  is computed by solving the set of  $N$  linear equations of the matrix form:

$$\underline{w} = (\underline{B} + \underline{O})^{-1} \underline{b}$$

$\underline{B}$  is the correlation matrix of SD increments between all pairs of observations

$\underline{b}$  is the correlation vector of pairs of observations and interpolation point  $k$

$\underline{O}$  is the matrix of SD observational errors (normalized by the first guess SD error variance) between all pairs of observations

# Brasnett (1999) Increment Correlation Functions

- ❖ Correlation coefficients  $\mu_{ij}$  between pairs of observations are computed as the product between the horizontal and vertical distance correlation functions:

$$\mu_{ij} = \alpha(r_{ij}) \beta(\Delta z_{ij})$$

$r_{ij}$  is the horizontal separation distance between observations

$\Delta z_{ij}$  is elevation (vertical) separation distance between observations

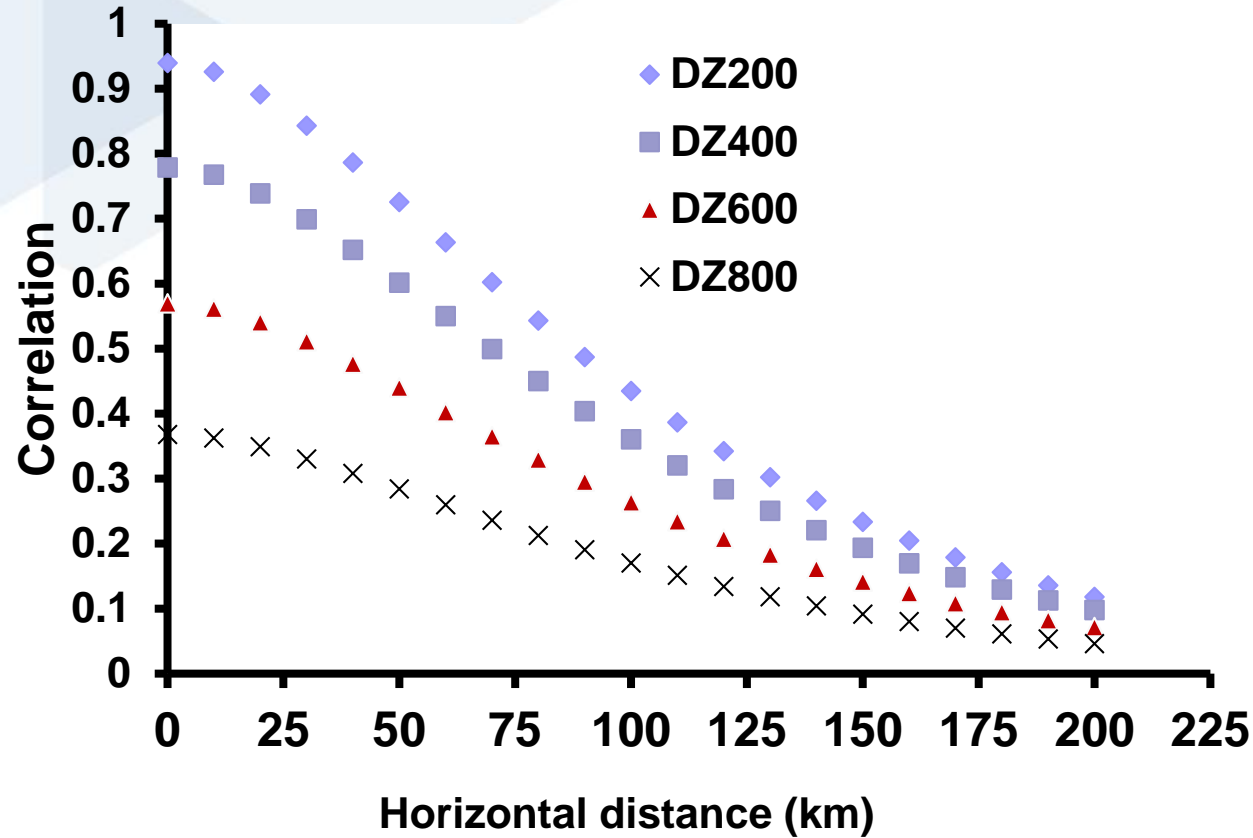
**2<sup>nd</sup> order autoregressive correlation function for horiz. distance**

$$\alpha(r_{ij}) = (1 + cr_{ij}) \exp(- cr_{ij}) \quad c = 0.018 \text{ km}^{-1} \quad (\text{e-folding scale} \approx 120 \text{ km})$$

**Square exponential correlation function for vertical distance**

$$\beta(\Delta z_{ij}) = \exp(- (\Delta z_{ij}/h)^2) \quad h = 800 \text{ m} \quad (\text{e-folding scale} = 800 \text{ m})$$

# Plots of spatial correlations as a function of horizontal distance for several vertical distance values



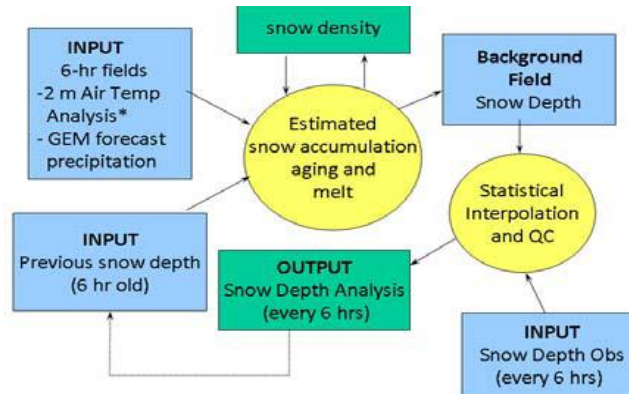


# Canadian Meteorological Center (CMC) Brasnett 1999 J. Applied. Meteorol, Global Snow Depth Analysis

## KEY FEATURES

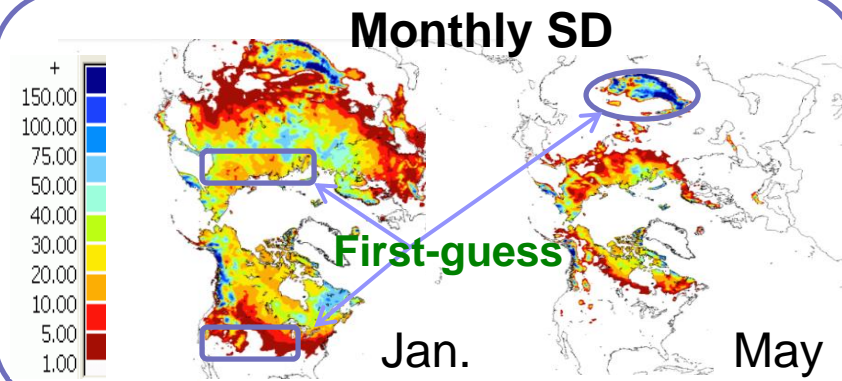
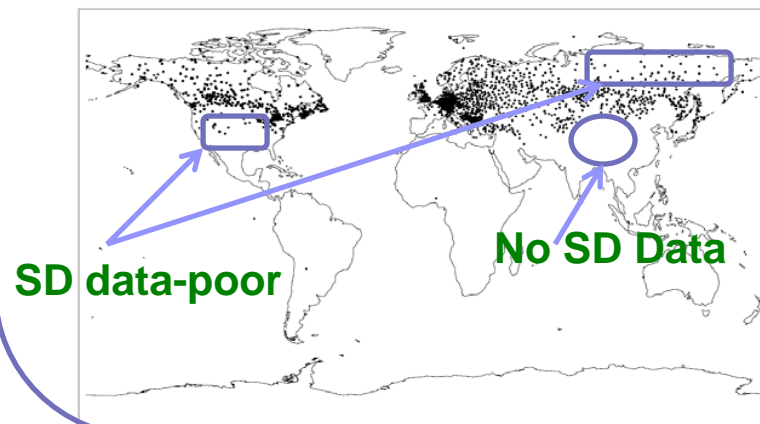
- ❖ 2-D Optimal Interpolation (OI) since March 1998, at 24-km & every 6 hours
- ❖ Initial guess - a simple snow accumulation and melt model using analyzed temperatures and forecast (six hour) precipitation from the CMC Global Environmental Multiscale (GEM) forecast
- ❖ Driven by in-situ SD observations; In regions where there are no SD observations, **analysis SD corresponds to the initial guess field.**

## Operational CMC SD Analysis Flowchart



\* Air temp corrected for difference in elevation between GEM model grid and snow depth analysis grid

## SYNOPSIS STATIONS



# Motivation for estimating spatial correlation structure

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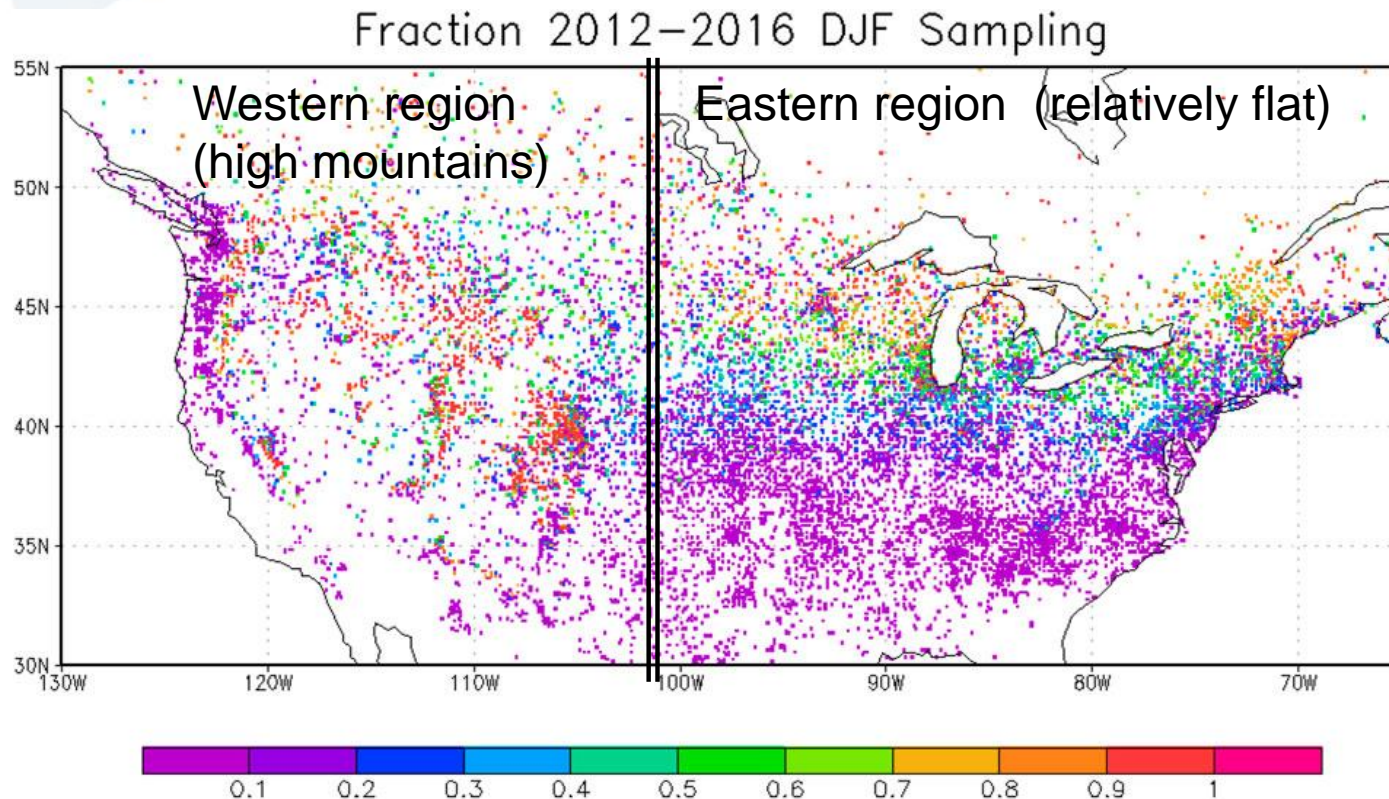
- ❖ No information provided of how well the correlation functions represent the structure of snow depth increments (*nowhere to be found in Gandin (1965) that the paper refers to*)
- ❖ *Method less beneficial over areas with stations separated by horizontal scales much larger than 120 km*

## New Study:

- ❖ 1: Estimate spatial correlations of measured snow depth and daily snow depth increments
- ❖ 2: Fit measured spatial correlations to modified Brasnett (1999) Equations to estimate correlation scale parameters

# 1. Method for computing observed correlations

- ❖ Eastern (horizontal correlations) and western (vertical correlations) sub-regions. In-situ stations resampled onto  $0.1 \times 0.1^\circ$  spatial grid. Binned horizontal correlations for 10 km from 0: 500 km range. Binned vertical correlations for 100 m from 0: 5000 m vertical range and restricted to adjacent grid cells.



## 2. Method for fitting binned correlations

❖ C1 is the autoregressive Brasnett equation – estimate  $\alpha$

$$C_1 = (1 + d\alpha_1) \exp(-d\alpha_1).$$

❖ C2 is similar to C1 but with an amplitude  $A$ , which represents correlation when separation distance approaches zero – estimate both  $A$  and  $\alpha$

$$C_2 = A_2 (1 + d\alpha_2) \exp(-d\alpha_2).$$

❖ C3 is the exponential brasnett equation, estimate both  $A$  and  $\alpha$

$$C_3 = A_3 \exp\left(- (d\alpha_3)^2\right).$$

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❖ *Least square method using data of measured correlation (C) and corresponding distances (d)*

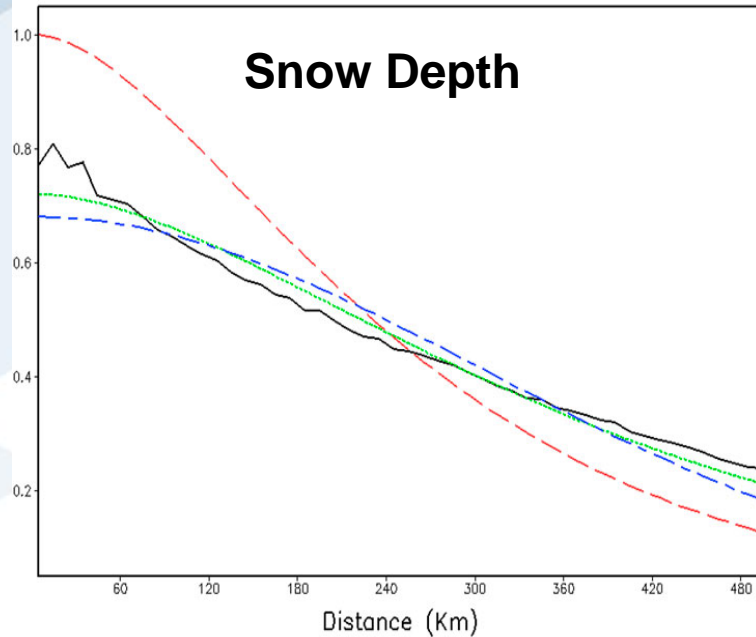
## Main Findings

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- **Large horizontal e-folding scales of observed snow depth and daily increments, at 430 km and 370 km, respectively, when including an amplitude, which is less than 1 (0.5 for daily increments and 0.7 for full snow depth).**
- **Without considering the amplitude, e-folding horizontal scales are severely underestimated and the fit is poor.**
- **Small vertical correlation of observed snow depth, at 460 m, but the vertical correlation of daily increments is much larger, at 1170 m**

# Results: Spatial correlations for horizontal distance

2012–2016 DJF Correlations

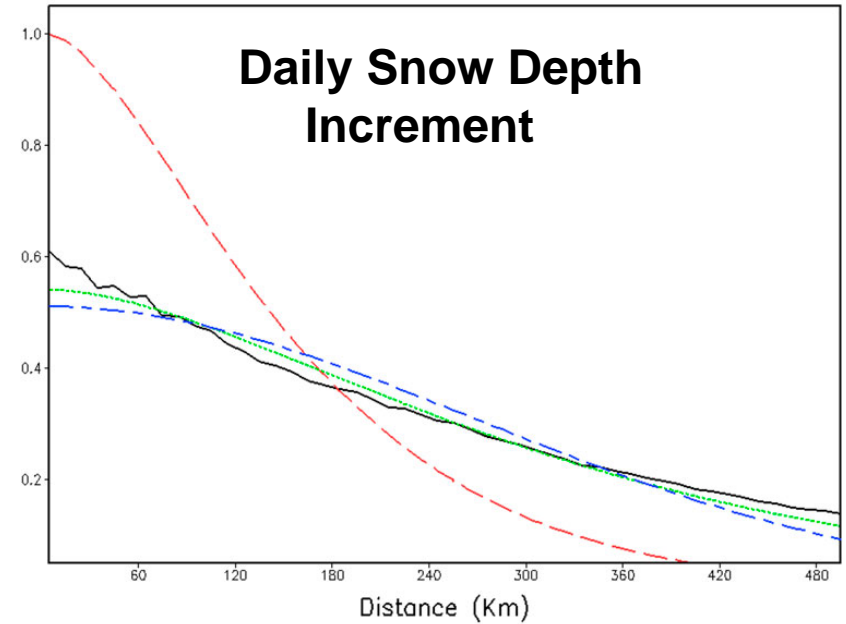


Corr      Fit1      Fit2      Fit3

	Scale	Amplitude	EFD	RMSE
Fit1	137.5	1.00 <sup>a</sup>	295.6	0.13
Fit2	200.0	0.72	430.1	0.03
Fit3	431.4	0.68	431.4	0.04

<sup>a</sup>Fixed a priori.

2012–2016 DJF Correlations



Corr      Fit1      Fit2      Fit3

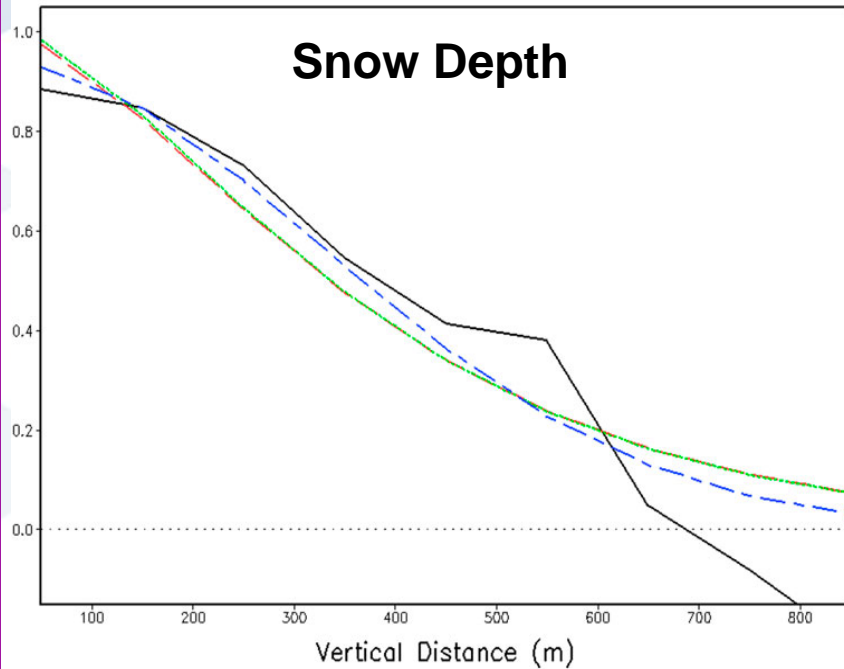
	Scale	Amplitude	EFD	RMSE
Fit1	84.6	1.00 <sup>a</sup>	181.9	0.18
Fit2	170.2	0.54	365.9	0.02
Fit3	378.0	0.51	378.0	0.03

<sup>a</sup>Fixed a tablef priori.

# Results: Spatial correlations for vertical distance

2012–2016 DJF Correlations

## Snow Depth



Corr

Fit1

Fit2

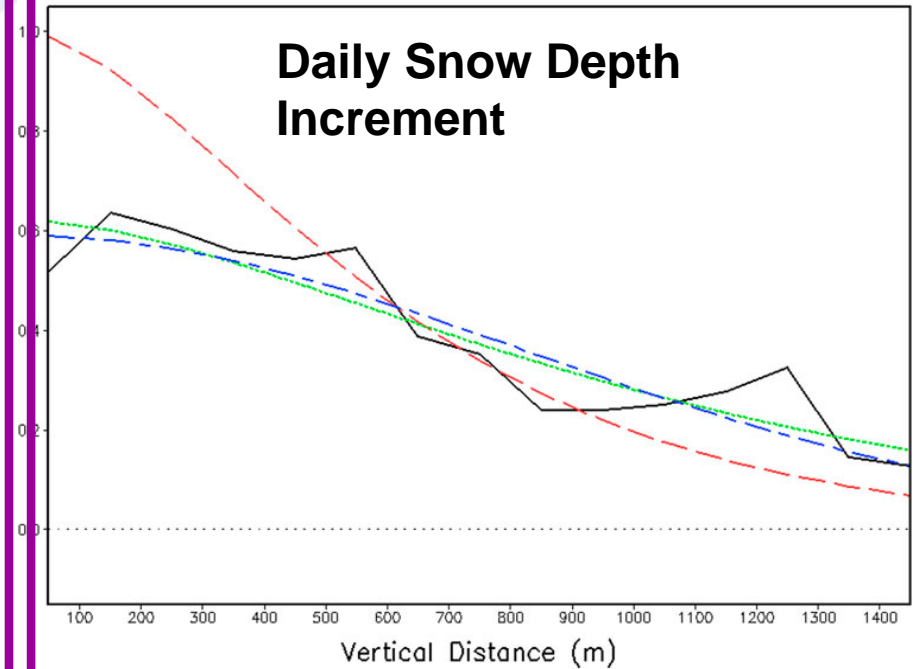
Fit3

	Scale	Amplitude	EFD	RMSE
Fit1	198.8	1.00*	427.5	0.14
Fit2	197.6	1.01	424.9	0.14
Fit3	461.1	0.94	461.1	0.12

\*Fixed a priori.

2012–2016 DJF Correlations

## Daily Snow Depth Increment



Corr

Fit1

Fit2

Fit3

	Scale	Amplitude	EFD	RMSE
Fit1	330.4	1.00*	710.4	0.18
Fit2	543.3	0.62	1,168.1	0.06
Fit3	1,166.7	0.59	1,166.7	0.06

\*Fixed a priori.

## Implications for operational snow depth analyses

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- **Horizontal correlation length scales of increments larger than 120 km need to be tested for improved predictions over poorly monitored areas.**
- **Vertical scales are more uncertain due to insufficient sampling over complex terrain and shorter vertical length scales.**
- **Interpolation of increments especially over complex terrain would be advantageous over full snow depth due to large scales and temporal consistency.**
- **Scales were assumed isotropic and fixed over the winter period, and need to be tested for seasonal adjustments.**